Dissipation of dark energy by cortex in knowledge retrieval

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We have devised a thermodynamic model of cortical neurodynamics in which we have incorporated the dissipation of so-called dark energy in knowledge retrieval as computed in the dissipative many-body model. We have extended the model first using the Carnot cycle to define our measures for energy and entropy, and then using the Rankine cycle to incorporate criticality and phase transitions. We describe the dynamics of two interactive fields of neural activity that express knowledge, one at high and the other at low energy density, and the two operators that create and annihilate the fields.

Keywords: axon; dendrite; cortex; electroencephalogram (EEG); dark energy; Carnot; phase transition; criticality; dissipative quantum model of brain, quantum dissipation; information; knowledge

PACS numbers:

I. INTRODUCTION

The aim of this paper is to study the energy consumption of the brain in the framework of the dissipative many-body model (Vitiello, 1995; Freeman and Vitiello, 2006) and the generalized Carnot cycle model (Freeman, Kozma and Vitiello, 2012). We focus our attention on the expenditure of energy to facilitate the emergence of patterns and dissipation of so-called dark energy in knowledge retrieval. The general picture of the process by which brains construct knowledge from information and how the generalized Carnot cycle describes it is presented in the following subsections. In Section II we consider the interplay between the macroscopically observed high energy need of the brain and the microscopic dynamics underlying the brain functional activity. Conclusions are presented in Section III. In the Appendix A the brain metabolic need of high energy density in conjunction with its dark energy dissipation is described. In Appendix B a brief summary of essential formal features of the dissipative many-body model is presented.

A. Knowledge, information and energy

Brains are thermodynamic systems that use chemical energy to construct knowledge from information (Freeman and Quian Quiroga, 2012). The oxidative metabolism of glucose provides the energy, as measured by oxygen depletion and carbon dioxide production. The sensory receptors in the body and on the body surface provide the information by absorbing energy of various types impinging from the internal and external environments (Laughlin et al., 1998). The role of each sensory receptor is selectively to convert a microscopic quantity of energy first to ionic currents (receptor potentials), then to a train of pulses (action potentials, units, spikes) on its axon. The sensory energy is weak (Barham, 1996), often a few molecules of scent, a few photons in a flash. Detection is facilitated by large arrays of equivalent receptors ($10^7 - 10^8$) for light, sound, vibration, and chemical substances. They form sheets on the body surfaces, which send bundles of axons into the brain. The pulse intervals, frequencies and delivery sites transmit the information by stages to the sensory cortices, where it is organized at high density as knowledge and accumulated over the lifetime of an individual as memories.

In an act of recognition a conditioned stimulus triggers an operator, a Hebbian nerve cell assembly, that abstracts, amplifies and generalizes to the category of a stimulus (Freeman, 2009). The assembly forms by repeated samples of information in reinforcement learning according to the Hebb rule: neurons that fire together wire together. The conditioned stimulus ignites the entire assembly, so the output signals the category of receptor and not the stimulus per se. The assembly provides the bolus of energy required to initiate a phase transition from a formless gas-like
phase of random activity to a structured liquid-like phase. The phase transition is by spontaneous breaking of the symmetry of the random phase (Freeman and Vitiello, 2006).

The fragment of knowledge from broken symmetry is expressed in two interactive fields of neural activity, which spread over the entire sensory cortex. The dendrites of the neurons generate a high-energy field of electric current that synchronizes cortical activity in a narrow-band oscillation. The knowledge content is expressed in a spatial pattern of amplitude modulation (AM). In the dissipative many-body model, these AM patterns are generated by the long range neuronal correlation sustained at microscopic level by the electric dipole quanta $A$ arising, in turn, as necessary consequence of the spontaneous symmetry breakdown (these quanta are the so-called Nambu–Goldstone (NG) quanta whose existence is predicted by the NG theorem in the hypothesis that spontaneous symmetry breakdown occurs; for an extended discussion on this point see Vitiello, 1995; 2001; Freeman and Vitiello, 2006; Blasone, Jizba and Vitiello, 2011). The axons generate a low-energy pulse cloud, a field of action potentials, which mediates the interactions among the dendrites in circular causality, thus counteracting the high-energy field of electric current synchronizing the cortical activity. Axonal pulse clouds find their counterpart at microscopic level in the field of $A$ quanta duplicating the quanta since they are coupled with dendritic current fields always in pairs. We thus have a double set of quantum modes, the $A$ and the $\tilde{A}$ quanta. The synchronization of the cortical activity and its in phase occurrence with the pulse cloud field are the manifestation of the many-body coherence of the $A - \tilde{A}$ condensate (see Section II). The pulse cloud is down-sampled to transmit cortical output needed for the central integration, which thus appears as the overall result of the microscopic many-body coherence. The oscillation is terminated (dissipates) after 3-5 cycles by an operator based in the bandwidth of the carrier frequency, thus requiring to incorporate dissipation (Vitiello, 1995) in the many-body model of brain (Umezawa and Ricciardi, 1967). This operator has been modeled using Rician statistics of extreme values in Freeman, 2009 (see Rice, 1950). We stress that in the dissipative many-body model, neurons, glia cells, dendrites, axons and other biological units are classical objects; the quantum variables are the ones associated to the quanta, $A$ and $\tilde{A}$, of the electric dipole field of the water molecules and other molecules.

B. Modeling knowledge construction with the Carnot cycle

The thermodynamic process of creating knowledge from information in a sensory cortex is cyclic. We have modeled the process using the generalized Carnot cycle (Freeman, Kozma and Vitiello, 2012), in which entropy is reduced by the expenditure of energy to facilitate the emergence of patterns (Fig. 1). We have observed the patterns in trained animals by using arrays of electrodes to record the electroencephalogram (EEG) from the surfaces of the visual, auditory, somatic and olfactory cortices as the animals respond to conditioned stimuli (Freeman, Walter, please insert a reference here****).

Each cycle has four steps. A cycle begins with a sensory cortex in a basal state of random background activity with low analytic power, $A^2(t)$, that is symmetric in having $1/f$ power spectral density (PSD) and no spatial or temporal pattern. The arrival of a stimulus-evoked sensory volley of pulses breaks the symmetry by initiating a narrow band oscillation that synchronizes the background pulse firings without increasing mean rates (isothermal compression), which causes a peak on the PSD and creates a spatial pattern of amplitude modulation (AM) in the EEG. We measured the AM patterns and deduced from the space-time changes the rate of entropy production, $D_e(t)$ (also information increase, $1/D_e(t)$). Each digitizing step increases the certainty of the AM pattern, hence adding increments of information.

In step two the AM pattern is fixed, and the analytic power continues to rise, not by synchronization but by expenditure of energy in transmission of the AM pattern (adiabatic heating), leading to maximal power and minimal entropy, with maximal classification of EEG patterns with respect to the conditioned stimuli containing information.

In step three the AM pattern dissolves as the firing rates diminish owing to the refractory periods of the neurons, and the strength of synaptic coupling waves (isothermal expansion). In step 4 the synchronization is annihilated as the distribution of characteristic frequencies in the PSD spectral peak go out of phase and cancel (adiabatic cooling). We define the area enclosed by the loop as a measure of pragmatic information (Atmanspacher and Scheingraber, 1990), which is the ratio of the rate of energy dissipation (power) to the rate of decrease in entropy (increase in information).

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1 The Hebb rule mentioned above, by which neurons that fire together wire together, thus appears as the macroscopic manifestation of the long range correlation NG modes generated in the microscopic process of spontaneous symmetry breakdown.
The activated knowledge in each cycle consists of the concomitant and coincident fields of activity, the high-energy field of dendritic currents (sustained at the microscopic level by the $A$ quanta) and the low-energy cloud of axonal pulses (sustained at the microscopic level by the $\tilde{A}$ quanta), respectively. Both are synchronized ($A - \tilde{A}$ coherence) in the same narrow band carrier frequency of the wave packet. The contents of the activated knowledge in each sensory cortex are observed and measured in the spatial AM pattern of the EEG manifesting the dendritic current density, which controls the intervals between pulses and thereby determines a pattern of pulse density in the cloud. The pulse cloud determines the current densities in circular causality, and it down-samples and transmits the information content of the AM pattern. Owing to sampling limitations the pattern in the cloud cannot be measured directly, so the conceptual contents are inferred from the macroscopic dendritic potentials (Ohl, Freeman and Scheich, 2001) and confirmed through ensemble averages of pulse trains from selected neurons (Quian Quiroga et al., 2007; Quian Quiroga, 2012).

We infer that the dendritic AM pattern is transmitted by the pulse cloud covering the whole of each sensory cortex. The memory bank and the sensory information exist at a microscopic level of single neurons and synapses, comparable to atoms and molecules and their attachment sites. The active memory is a macroscopic collective field of energy that is sustained by the interactions of million of neurons and billions of synapses. We postulate that the background activity prior to a stimulus trigger is sparse, random firing of neurons in low-density comparable to a gas-like phase, and that the recollection of a memory fragment forms by a phase transition of the cortex as it condenses to a patterned liquid-like phase. After transmission of the wave packet the activity evaporates (dissipates), and the cortex irretrievably annihilates the initiating information and its context.

The cycle is exceedingly energy-intensive, almost equally in the two alternating phases of reception and transmission. The metabolic processes involved in the energy collection and consumption are described in Appendix A. In the following we focus our attention on the energy requirements for criticality and phase transitions. Indeed, a further demand for energy is imposed by the requirement for background activity in living brains. The steady state random activity is governed by an (***delete: a non-zero ) attractor, which is sustained by the positive feedback by which the excitatory neurons continually excite each other, restrained from excessive firing by their refractory periods (Freeman and Zhai, 2009). The mutual excitation maintains a steady state of criticality (Freeman and Quian Quiroga, 2012) far from thermodynamic equilibrium (Fig. 2). Criticality sustains a high-energy state of readiness for phase transition to meet the exigencies of an unpredictable and dangerous world, which adds to the losses of energy by leakage, thus constantly dissipating energy in self-sustained clouds of axonal action potentials and dendritic currents. The many-body dissipative model allows us to compute the condensate energy supplying the energy requirements for criticality and phase transitions. Thus we present such a computation in the following.

Essential aspects of the formalism of the dissipative model are very briefly summarized in Appendix B (details can

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**II. MACROSCOPIC FIELDS OF MICROSCOPIC NEURAL ACTIVITY**

![Diagram of a Carnot cycle](image)

**FIG. 1:** A. An ideal Carnot cycle is illustrated with four isoclines for a heat engine. B. The upper inset illustrates a sequence of three Carnot cycles carrying AM patterns in the superimposed filtered ECoGs from 64 electrodes. The variables representing temperature and entropy are replaced with indices derived from multichannel EEG recording. We start the cycle (1) at minimal mean power, $A^2(t)$, and maximal chaotic disorder, $D_e(t)$. At the other extreme (3) the power and information are maximal (the disorder is minimal). The knowledge increment is estimated from the area in the rectangle, which defines the pragmatic information (Atmanspacher and Scheingraber, 1990). From Freeman and Quian Quiroga (2012)
be found in Vitiello, 1995; Freeman and Vitiello, 2006). In the notation there introduced, $|0\rangle$ and $|0(\theta)\rangle_N$ denote states of the brain activity corresponding to the absence of quanta $A_k$ and $\bar{A}_k$ (the vacuum $|0\rangle$) and the condensate state of condensation density $N_{A_k}$ (and $N_{\bar{A}_k}$), respectively.

We observe that for the inner products (the overlaps) of these states the relations hold: $\lim_{N\to\infty} N\langle0(\theta)|0(\theta)\rangle_N = 0$ and $\lim_{N\to\infty} N\langle0(\theta)|0(\theta')\rangle_N = 0$, for any $N$ and any $N' \neq N$. These relations signal that in the limit of infinitely many degrees of freedom (the infinite volume limit of quantum field theory) the process of condensation of the $A$ and $\bar{A}$ modes is a phase transition process: the states $|0\rangle$ and $|0(\theta)\rangle_N$, for any $N$, represent 'distinct' phases in the brain activity since no overlap exist among them for different values of $N$. These relations also express the criticality present in the brain background activity since their meaning is that no unitary transformation exists able to lead from one phase coded by $N$ to another phase coded by $N'$, with $N' \neq N$: they are ‘unitarily inequivalent phases’ and transitions from phase to phase are critical transition processes. The criticality here identified is characterized by chaotic, scale-free behavior and power laws, as indeed laboratory observations show (Freeman...Refs ???), which are features implied by the model since phase transition processes turn out to be described by chaotic “trajectories” from phase to phase (Vitiello, 2004), and coherent states such as $|0(\theta)\rangle_N$ have been shown to be characterized by scale-free, power law features (Vitiello, 2009a; 2009b; 2012).

Next, the condensate density in the state $|0(\theta,t)\rangle_N$, the time evolved at time $t$ of $|0(\theta)\rangle_N$, is given by (Vitiello, 1995)

$$N_{A_k}(t) = N\langle0(\theta,t)|A_k^\dagger A_k|0(\theta,t)\rangle_N = \sinh^2(\Gamma_k t + \theta_k) = \frac{1}{e^{\beta(t)}E_k - 1},$$

(1)

where $\beta(t)$ is assumed to be slowly varying in time in order to ensure that quasi-stationary conditions be satisfied at any time $t$. From the minimization of the free energy $F_A$ of the $A$ quanta, $dF_A = dE_A - (1/\beta) dS_A = 0$ one recognizes that $E_A = \int d^3 k \omega_k N_{A_k}$ is the internal energy of the system. $S_A$ is the entropy and as usual heat is $dQ = dS_A/\beta$. Thus the change in time of condensate, $dN_{A_k}/dt$, turns out into heat dissipation $dQ$. $dF_A = 0$ also expresses the first principle of thermodynamics for a system coupled with environment at constant temperature and in absence of mechanical work. The non-equilibrium dynamics has been studied in detail by use of the Ginzburg-Landau time dependent equation in Freeman, Livi, Obinata and Vitiello, 2012. In any case it happens that for any $t \neq t'$ it is $\lim_{\tau \to \infty} N\langle0(\theta,t)|0(\theta,t')\rangle_N = 0$, which shows that brain activity is far from the equilibrium and characterized by criticality at any time $t$. Time evolution of the phase coded by same $N$ appears to be a far from the equilibrium critical process $\langle0(\theta,t)\rangle_N$ and $\langle0(\theta,t')\rangle_{N'}$, even with same $N$, are unitarily inequivalent phases.

We finally compute the energy contribution of the (vacuum) condensation density (1) (the brain ‘dark energy’) to the energy requirements for criticality and phase transitions.

In a standard fashion, the energy density of the condensate, at a given time $t$, is given by

$$\rho = \int d^3 k \omega_k N\langle0(\theta,t)| : A_k^\dagger A_k : |0(\theta,t)\rangle_N$$

(2)
where \( \ldots \) : denotes the normal ordering with respect to the vacuum \( |0\rangle \). Then

\[
\rho = 4\pi^2 \int_0^\infty dk \, k^2 \omega_k \sinh^2(\Gamma_k t + \theta_k) = 4\pi^2 \int_0^\infty dk \, k^2 \frac{\omega_k}{e^{\beta(E_k)} - 1}.
\] (3)

For massless fields, such as NG fields, \( \omega_k = k \), the above integral becomes (for \( h = c = k_B = 1 \))

\[
\rho_{m=0} = 4\pi^2 \int_0^\infty dk \, k^3 \frac{k^3}{e^{\beta(E_k)} - 1} = \frac{4}{15} \pi^6 T^4(t).
\] (4)

Boundary effects and impurities, however, may contribute to give a non-vanishing effective mass to the NG fields (Freeman and Vitiello, 2010; Blasone, Jizba and Vitiello, 2011). In such a case of massive fields, the integral in Eq.(3) can be solved numerically. It converges and its upper bound is given by Eq.(4).

From our result it appears that a large amount of energy is stored in the brain background activity state. It constitutes the reservoir for the brain energy requirements for criticality and phase transitions. It may be considered to be the reservoir for energy exchanges in the Carnot-like cycle discussed above.

III. DISCUSSION AND CONCLUSION

The property of brains we address in this report is the complexity of associations that we experience in flashes of recognition in recall induced by sensory stimuli. The richness of detail is explained by three outstanding structural features of the neuropil. One is the high divergence and convergence among axons and dendrites in neuropil, with \( 1/f \) distributions of connection distances that facilitate scale-free neurodynamics; another is the extreme packing density of cells and fibers, which brings enormous numbers of neurons and synapses within the correlation range for synchronization of neuronal activity; a third is the maintenance of the neuropil in a state of criticality, a readiness for abrupt change by phase transitions from expectancy to realization and back again repeatedly in tracking changes in the environment.

These features can readily explain the observations of the dark energy at high density and rate of dissipation by neuropil, which have their greatest values in the human brain at the pinnacle of biological intelligence. What is to be modeled and explained is the mobilization of the myriad microscopic details that are stored in the modified synapses among the interconnected neurons into the macroscopic order that is expressed by the pulse cloud and its controlling field of dendritic currents, which we observe in the EEG and experience in flashes of insight.

What is unclear is the extent to which the active states established in excitatory and inhibitory neurons spread into the tips of active dendrites. On the one hand the fine shafts and tips may only support the growth cones for future addition of area to the dendritic trees for new synapses with no contributions to interactions. On the other hand they may interact by ephapses to the ionic currents from their neighbors densely packed in the neuropil. Ephaptic transmission has been thoroughly documented to occur in the substantia gelatinosa, a form of laminated neuropil that resembles cortex in the sensory grey matter of the spinal cord (Grundfest, 1954; Brooks and Koizami, 1956). It occurs only with states of intense excitation or extreme sensitivity, such as in pain (Wall and Lidierth, 1997), so it is generally regarded as pathological. It has also been reported to mediate remarkable increases in synchronization of neural firing in vivo in cortical slices (Anastassiou et al., 2011).

Our data from EEG recording suggest that ephapses may play a role in the phase transition by which the low-density background activity rapidly changes to high-density activity with synchronization with emergence of an AM pattern, provided that the cortex is in criticality, and that there is a source of a bolus of transition energy. Our thermodynamic model is based on a generalization of the Carnot cycle, the Rankine cycle (Fig. 2), which is embedded in a critical domain, such that with increasing density of interactions the neural activity condenses into the liquid-like phase, and it evaporates by uncoupling into the gas-like phase. In the dissipative many-body modeling of brain, we argue that the condensation requires an operator that creates the AM pattern by sequestering a large quantity of energy in the high-density state, and that the energy is relinquished as heat on return to the basal state through annihilation of the AM pattern by a second operator. It is these operators that bring microscopic information stored in memory to be the reservoir for energy exchanges in the Carnot-like cycle discussed above.
No one knows exactly how ephaptic transmission works, because the packing density is too great to allow use of existent electrophysiological methods. Candidate mechanisms may include coupling through water dipoles (Freeman and Vitiello, 2010). The condensate of dipole wave quanta $A$ and $\tilde{A}$ may indeed provide the proper dynamical environment and the necessary energy to promote direct, non-synaptic, point coupling between neighboring axons and dendrites. Ephaptic coupling and transmission among fiber terminals may then result to be another macroscopic manifestation of the microscopic many-body dynamics. The computation of the energy contribution coming from the brain background microscopic activity state presented in the present paper may represent a step forward in the understanding of the ephaptic coupling.

**APPENDIX A: BRAIN DARK ENERGY AND THE NEED FOR HIGH ENERGY DENSITY**

In comparison to other tissues the human brain has 2% of body mass but dissipates 20% – 25% of resting energy (2% – 8% in most vertebrates, 10% – 15% in lesser primates), as measured by oxygen depletion in the venous return from the brain, compared with total body consumption of oxygen. Imaging of blood oxygen levels of depletion (BOLD) with $O^{15}$ and positron emission tomography (PET) in various regions has shown that the neurons in cerebral cortex are most demanding (Attwell and Laughlin, 2001), followed in descending order by those in basal ganglia, brain stem and white matter (Marchal et al., 1992; Karbowski, 2007; Harris and Attwell, 2011), which have rates comparable to those for heart, liver and kidney, still above the resting rates for most tissues of the body. The ten-fold discrepancy for human cortex above the whole body has led to the sobriquet dark energy, in analogy to dark matter in astrophysics (Raichle and Gusnard, 2002).

The high energy density and dissipation are imposed by the immense connectivity among neurons that is required to interconnect manifold information into knowledge at high density. What most distinguishes neurons from other cells is their unique shape. The long axonal filaments with multiple branches provide the basis for the strong divergence from each neuron to $\sim 10^5$ other neurons (Braitenberg and Schz, 1998). The radiating dendritic trees provide the immense surface area required for the converging synapses from roughly $10^4$ other neurons, each of which must have a current path to the trigger zone. The density of cells is exceedingly high: $10^3/mm^3$. The embedding glial cells outnumber the neurons 10 : 1 in humans and contribute their filaments to the neuropil. The cell bodies of most cortical neurons are aligned in layers, and the dendrites and axons tend to be oriented parallel or perpendicular to the cortical surface, often in bundles (Innocenti and Vercelli, 2010). The cortex is a thin layer, in humans $2-4\,mm$ in depth and $2-3\times10^5\,mm$ in area. The resulting small volume compared to the large area drives the wrinkling of the cortical surface into the gyri and sulci. A network of capillaries provides nutrients and $O_2$ and removes $CO_2$; it also provides a low resistance extracellular path for dendritic currents. The dense mix of filaments in cortical tissue resembles the pile of a rug; hence the anatomical term: laminated neuropil.

The fineness of compartmentalization of the neuropil by neural membranes of the fibers far exceeds that of any other tissue. Most filaments are so small that they can only be seen by electron microscopy; the minimum diameter is 0.1 micron; the median diameter in cross-section being 0.11 – 0.2 micron (Olives, Montiel and Aboitz, 2001). To receive and transmit information each fiber maintains the transmembrane gradients of $Na^+$ and $K^+$ required for the resting membrane potential by ionic pumps that are fueled by ATP from mitochondria. The rate of passive leakage of $Na^+$ and $K^+$ ions is proportional to the surface areas of the external membranes encasing the neurons and of passive $H^+$ ions due to the gradients across the internal membranes of the mitochondria. Hence the ultrastructure of cortex required for dense connectivity imposes intensive dissipation of chemical energy by brains both at rest and engaged in cognition.

According to the membrane pacemaker theory of metabolism (Kleiber, 1975), the differences in metabolism between cerebral and non-cerebral tissues are caused by differences both in membrane chemical composition and in ionic pumps. The polyunsaturated fatty acids that are characteristic of neuronal membranes have distinctive physical properties of flexibility that cause the proteins in the membranes such as the voltage-dependent ionic channels to have high molecular activity (Hubert and Else, 2005), which results in higher rates of metabolism. The long axons impose communication by $Na^+$ action potentials, so that only in neuropil (and to a lesser extent in kidney) does $Na^+/K^+$-ATPase dominate energy production from glucose (Astrup, Srensen PM and Srensen HR, 1981; Atwell and Laughlin, 2001), constituting 40% – 47% of brain basal metabolic rate. Only neuropil has the supportive glial cells maintaining the blood-brain barrier, which sustains the necessary constancy of the fluid environment surrounding neurons by myriad biochemical reactions that cost energy, which cannot be estimated reliably, so inseparable are neurons and glia in normal function (Magistretti, 2006). Only neurons sum excitatory and inhibitory synaptic potentials with subtraction to near-zero voltage, but with addition of the costs in metabolic energy.
APPENDIX B: ESSENTIAL FORMAL FEATURES OF THE DISSIPATIVE MANY-BODY MODEL

A full account of the dissipative many-body model of brain and its formalism is presented in Vitiello, 1995; Vitiello, 2001; Freeman and Vitiello, 2006. Here we only present some formal features finalized to a better understanding of some remarks in the main text and in order to introduce notations there used.

As mentioned in subsection I.A, the NG quanta \( A_k \) and their condensation in the ground state are dynamically generated through the spontaneous breakdown of the symmetry triggered by the external stimuli. The NG theorem predicts that the NG quanta or modes have zero mass and thus they can span the whole system volume without inertia. Moreover, in the quantum formalism, to each quantum or particle one may associate a corresponding wave, and vice-versa. So, we may also think of the \( A_k \)'s as the quanta of the (electric) dipole wave spanning the whole system. The NG quanta thus appear as collective modes since they express the collective dynamical interaction among the system microscopic constituents in the regime of symmetry breakdown. Again, we stress that in the dissipative many-body model the microscopic quantum constituents are the electric dipole field quanta associated to the quantum fluctuations of the electric dipole field of water molecules and other molecule in the system. Neurons, glia cells, dendrites, axons and other biological units are classical objects.

By considering also the mirror quanta \( \tilde{A}_k \) (cf. subsection I.A), let \(|0\rangle\) denote the state without \( A_k \) and \( \tilde{A}_k \) quanta (the vacuum state for the \( A_k \) and \( \tilde{A}_k \) quanta), namely the state which is annihilated by \( A_k \) and \( \tilde{A}_k \). \( A_k |0\rangle = \tilde{A}_k |0\rangle = 0 \), for any \( k \), \(|0\rangle \equiv |N_{A_k} = 0, N_{\tilde{A}_k} = 0\rangle \), with \( N_{A_k} \) and \( N_{\tilde{A}_k} \) denoting the number of \( A_k \) and \( \tilde{A}_k \), respectively. We use for simplicity only one index \( k \) specifying the quantum numbers, e.g. the momentum, of the \( A \) and \( \tilde{A} \) operators.

The dynamical generation and the condensation of the NG modes, say at time \( t_0 = 0 \), leads to the new state \(|\theta(t)\rangle_N \equiv |N_{A_k} = N_{\tilde{A}_k} = 0, \forall k\rangle \), with \( N \equiv \{N_{A_k} = N_{\tilde{A}_k} = 0, \forall k, \at t_0 = 0\} \).

Note that equal numbers (couples) of \( A_k \) and \( \tilde{A}_k \), for any \( k \), are condensed in \(|\theta(0)\rangle_N \). Since the NG modes (and their mirror modes \( \tilde{A} \)) are massless, the condensation of a number of them with vanishing momentum in the system ground state does not add energy to such a state. Thus, (ininitely) many condensed \( \theta \)-states \(|\theta(t)\rangle_N \), degenerate in the energy, are obtained, differing among themselves solely for the different value of the number \( N \) of condensed \( A_k \) and \( \tilde{A}_k \) couples: \( N \) acts as a code labeling these degenerate states.

At finite volume \( V \), \(|\theta(t)\rangle_N \) turns out to be a two-mode coherent state and can be represented as \(|\theta(t)\rangle_N = \exp(-\frac{i}{\hbar}\theta(t))|\theta(t)\rangle_N \) with \( G(\theta(t)) = -i \sum_k \theta_k (A_k^\dagger \tilde{A}_k^\dagger - A_k \tilde{A}_k) \). Here \( A_k^\dagger \) and \( \tilde{A}_k^\dagger \) denote the creation operators.

Through Bogoliubov transformations, one may introduce also the annihilation (and creation) operators \( A_k(\theta_k) \) and \( \tilde{A}_k(\theta_k) \) for the state \(|\theta(t)\rangle_N \). By minimizing the free energy functional for such a state one then finds the condensation density \( N(\theta)|A_k^\dagger A_k|\theta(t)\rangle_N = N_{A_k} = \sinh^2 \theta_k = 1/(e^{\beta E_k} - 1) \) (and similar expression for \( N_{\tilde{A}_k} \)), namely the Bose-Einstein distribution for \( A_k \) (and \( \tilde{A}_k \)). Here \( \beta = 1/T \), with \( T \) the temperature, and \( E_k \equiv \omega_k \) (we use the Boltzmann constant \( K_B = 1 \) and \( \hbar = 1 = c \)). This relation shows the link between the condensate density (and thus the code \( N \)) and the set of \( \theta_k \) parameters. Sets made by different \( \theta_k \) values correspond to different condensation densities (different codes \( N \), different ways or degrees of symmetry breaking, different memories).

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