Quantum mechanics and gravitation: a difficult merging.

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General Relativity, or more precisely Geometrodynamics, is the successful theory for gravitational phenomena, including many aspects of cosmology. The recent spectacular results, about the existence of gravitational waves proven by the large improved interferometric apparatuses of the Ligo and Virgo teams, confirm the soundness of General Relativity.

However, gravitation is a basic natural interaction quite distinct from the other interactions, the strong, the weak and the electromagnetic. All of these find a coherent
theoretical interpretation in the frame of the quantum field standard model of elementary particles. On the other end the full merging of gravitation with quantum mechanics has not been achieved yet, even though in some sectors very brilliant results do exist, as we have seen for example in the talk by Carlo Rovelli. On the other hand, convincing evidence of possible physical signatures of quantum gravity has been provided in recent time, as for example in the work of Giovanni Amelino-Camelia.
The purpose of this talk is to recall some features, and possible difficulties, of the merging of gravitation and quantum mechanics. Emphasis will be given to the main conceptual aspects, without the exploitation of heavy technical details.

Therefore, we will follow a kind of itinerary organized in the following four steps.

- the basic structure of classical mechanics, classical electrodynamics, and classical geometrodynamics.
- the creation of modern quantum mechanics

- quantum mechanics and quantum field theory (Göttingen)

- the route to quantum geometrodynamics
Classical mechanics is a very powerful general theoretical scheme ruling all mechanical phenomena, with some important limitations, which have been completely clarified only after the advent of special relativity and quantum mechanics. It is a quite modern science (compared for example with Geometry) developed through the efforts of Galileo Galilei, Isaac Newton, and many other people (Lagrange, Hamilton, ....).
In the simple case of a system made by $N$ particles, the “state” of the system at each time is given by the positions and the velocities (or equivalently momenta) of all particles. The time evolution is ruled by Newton second principle of dynamics expressing the accelerations on each particle through forces, which are in principle measurable in themselves.

This scheme is extremely productive. For example, it provides a model for the Solar system, where the points are the Sun and the
other planets, and the forces acting on each particle are given by the Newtonian gravitational attraction. To give an idea of the effectiveness, let us recall that it was possible to calculate that the perihelion of the planet Mercury should advance of 531 arc-second per century, due to the gravitational effects of Venus, Earth, Jupiter (as a matter of fact the observed value is 574, the missing value 43 is due to general relativity, but this is another story).

Mechanics is usefully complemented by electrodynamics. Here the “state” at each time
is given by “fields” (in principle measurable), defined at all space-time points \((\vec{x}, t)\): an electric field \(E(\vec{x}, t)\), and a magnetic field \(B(\vec{x}, t)\). These fields evolve in time according to the Maxwell equations, which is not necessary to write explicitly here. The sources of the electromagnetic fields are given by electric charges and currents. On the other hand, the electromagnetic field will act on charged particles according to the Lorentz force.

Maxwell equations allow the existence of electromagnetic waves travelling in vacuum with
a speed $c$. According to the frequency, these waves represent visible light, infrared, ultra-violet, radio waves, etc. All these phenomena are unified and described by a single set of equations.

Geometrodynamics gives a curvature to space-time, described by a metric tensor described by a set of parameters $g_{\mu \nu}$. The evolution is ruled by the Hilbert-Einstein equations, which were originally found by David Hilbert through a variational principle. Einstein equations tell how curvature is ruled by matter,
and how matter evolves on the curved space. The equations are highly nonlinear. The components of the metric tensor have a partial gauge nature, since the equations of motions are invariant under any coordinate diffeomorphism. The equations allow the existence of gravitation waves travelling in space with a speed $c$. 
Albert Einstein (1879-1955)
David Hilbert (1862-1943)
Now we go to the years 1925-1930 when there was an impressive development leading to the “creation” of modern quantum mechanics.

**Helgoland and Göttingen**

Let us start from Helgoland (Sacred Island), a small island in the North Sea.
In the Spring 1925, the young Werner Heisenberg (1901-1976), a former student of Niels Bohr, took refuge in the island, as a relieve against a strong hay fever. At that time, the island was almost void of vegetation, and therefore free of allergenic pollens.
quantum mechanics and quantum field theory

During the stay in Helgoland and the subsequent return to Göttingen, all the most important problems, related to the Old Quantum Mechanics of Bohr-Sommerfeld and to the wave-particle dilemma for light, were conceptually solved essentially in one stroke by the “creation” of the general theory of modern quantum mechanics, achieved by Werner Heisenberg in his 1925 Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen (Quantum-Theoretical
Reinterpretation of Kinematical and Mechanical Relations), and further developed by Max Born, Pascual Jordan and Heisenberg himself.
Werner Heisenberg (1901-1976)
Max Born (1882-1970)
Pascual Jordan (1902-1980)
Heisenberg is awarded the 1932 Nobel Prize in Physics (actually announced next year 1933), “for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen.” Born must wait until 1954, “for his fundamental research in quantum mechanics, especially for his statistical interpretation of the wavefunction”. Jordan, whose scientific merits are obtaining more and more deep credit along the years, will never get this recognition, for very well known reasons.
Über quantentheoretische Umdeutung
kinematischer und mechanischer Beziehungen.

Von W. Heisenberg in Göttingen.

(Eingegangen am 29. Juli 1925.)

In der Arbeit soll versucht werden, Grundlagen zu gewinnen für eine quantentheoretische Mechanik, die ausschließlich auf Beziehungen zwischen prinzipiell beobachtbaren Größen basiert.

Bekanntlich läßt sich gegen die formalen Regeln, die allgemein in der Quantentheorie zur Berechnung beobachtbarer Größen (z. B. der Energie im Wasserstoffatom) benutzt werden, der schwerwiegende Einwand erheben, daß jene Regeln als wesentlichen Bestandteil Beziehungen enthalten zwischen Größen, die scheinbar prinzipiell nicht beobachtet werden können (wie z. B. Ort, Umlaufzeit des Elektrons), daß also jenen Regeln offenbar jedes anschauliche physikalische Fundament mangelt, wenn man nicht immer noch an der Hoffnung festhalten will, daß jene bis jetzt unbeobachtbaren Größen später vielleicht experimentell zugänglich gemacht werden könnten. Diese Hoffnung könnte als berechtigt angesehen werden, wenn die genannten Regeln in sich konsequent und auf einen bestimmten umgrenzten Bereich quantentheoretischer Probleme anwendbar wären. Die Erfahrung zeigt aber, daß sich nur das Wasserstoffatom und der Starkeffekt dieses Atoms jenen formalen Regeln der Quantentheorie fügen, daß aber schon beim Problem der „gekreuzten Felder" (Wasserstoffatom in elektrischem und magnetischem Feld verschiedener Richtung) fundamentale Schwierigkeiten auftreten, daß die Reaktion der Atome auf periodisch wechselnde Felder sicherlich nicht durch die genannten Regeln beschrieben werden kann, und daß schließlich eine Ausdehnung der Quantenregeln auf die Behandlung der Atome mit mehreren Elektronen sich als unmöglich erwiesen hat. Es ist üblich geworden, dieses Versagen der quantentheoretischen Regeln, die ja wesentlich durch die Anwendung der klassischen Mechanik charakterisiert waren, als Abweichung von der klassischen Mechanik zu bezeichnen. Diese Bezeichnung kann aber wohl kaum als sinngemäß angesehen werden, wenn man bedenkt, daß schon die (ja ganz allgemein gültige) Einstein-Bohrsche Frequenzbedingung eine so völlige Abweichung von der klassischen Mechanik oder besser, vom Standpunkt der Wellenmechanik, nicht darstellt, daß auch bei den einfachsten quantentheoretischen Problemen an

Zeitschrift für Physik. Bd. XXXIII.
The Heisenberg *Umdeutung* (reinterpretation) is deeply motivated by the decision to found theoretical quantum mechanics exclusively upon relations between quantities which in principle are observable, by avoiding the *ad hoc* constructions of Old Quantum Mechanics, based for example on special constraints on the possible ORBITS of the electron in the Hydrogen atom.

The proposal is incredibly simple. Just as in classical mechanics, the basic observables
are position and momentum, and the equations of motion are the same as in classical mechanics, which we rewrite here for a one-dimensional system

\[ \frac{d}{dt} x(t) = \frac{p(t)}{m}, \quad \frac{d}{dt} p(t) = F(x(t), p(t)). \]

However, and here is the *Undeutung* (reinterpretation), position and momentum are not just assumed to be numbers, as in classical mechanics, but they are assumed to be infinite-dimensional complex Hermitian matrices, obeying appropriate commutation rules,
which in the one-dimensional case are written as

\[ xp - px = i\hbar, \]

where \( i \) is the imaginary unit, and \( \hbar \) is Planck’s constant scaled by \( 2\pi \), \( \hbar = \hbar/2\pi \).

From these very simple assumptions, it is possible to derive all quantum mechanical physical content, in a context much larger than the old quantum mechanics. Energy of course is given by the matrix

\[ H = \frac{p^2}{2m} + V(q), \]
where \( V(q) \) is the potential giving rise to the force. The energy levels are the eigenvalues of the energy matrix.

The scheme is very general and allows deep and large applications. The first trouble is solved: to go beyond the Old Quantum Theory.

Moreover, along the same line, also the dilemma wave-particle for the photon is also solved, in the 1925 paper called *Dreimännerarbeit* (the work of the three men (3M): Born, Heisenberg, Jordan).
Zur Quantenmechanik. II.
Von M. Born, W. Heisenberg und P. Jordan in Göttingen.
(Eingegangen am 16. November 1925.)

Die aus Heisenbergs Ansätzen in Teil I dieser Arbeit entwickelte Quanten-
mechanik wird auf Systeme von beliebig vielen Freiheitsgraden ausgedehnt. Die
Störungstheorie wird für nicht entartete und eine große Klasse entarteter Systeme
durchgeführt und ihr Zusammenhang mit der Eigenwerttheorie Hermitescher Formen
nachgewiesen. Die gewonnenen Resultate werden zur Ableitung der Sätze über
Impuls und Drehimpuls und zur Ableitung von Auswahlregeln und Intensitäts-
formeln benutzt. Schließlich werden die Ansätze der Theorie auf die Statistik
der Eigenschwingungen eines Hohlraumes angewendet.

Einleitung. Die vorliegende Arbeit versucht den weiteren Ausbau
der Theorie einer allgemeinen quantenmechanischen Mechanik, deren
physikalische und mathematische Grundlagen in zwei vorausgegangenen
Arbeiten der Verfasser1) dargestellt sind. Es erwies sich als möglich, die
benannte Theorie auf Systeme von mehreren Freiheitsgraden zu er-
weitern2) (Kap. 2) und durch Einführung der „kanonischen Transformationen“ das Problem der Integration der Bewegungsgleichungen auf be-
kannnte mathematische Fragestellungen zurückzuführen; dabei ergab sich
mittels dieser Theorie der kanonischen Transformationen einerseits eine
Störungstheorie (Kap. 1, § 4), die eine weitgehende Ähnlichkeit mit der
der klassischen Störungstheorie aufweist, andererseits ein Zusammenhang der
Quantenmechanik mit der mathematisch so hochentwickelten Theorie der
quadrischen Formen unendlich vieler Variablen (Kap. 3). — Bevor wir
aber auf die Darstellung dieser weiteren Entwicklung der Theorie eingehen,
werden wir ihren physikalischen Inhalt genauer zu umgrenzen suchen.

Der Ausgangspunkt der versuchten Theorie war die Überzeugung, daß es nicht möglich sein werde, der Schwierigkeiten, die uns in der
Quantenmechanik gerade in den letzten Jahren auf Schritt und Tritt be-
gegneten, Herr zu werden, ehe für die Mechanik der Atom- und Elek-
tronenbewegungen ein mathematisches System von Beziehungen zwischen
prinzipiell beobachtbaren Größen zur Verfügung stünde von ähnlicher

1) W. Heisenberg, ZS. f. Phys. 33, 879, 1925. M. Born und P. Jordan,
ZS. f. Phys. 34, 868, 1925. Im folgenden als (Teil I) zitiert.
2) Anm. bei der Korr. In einer inzwischen erschienenen Arbeit von P. Dirac
in dieser Arbeit enthaltenen Gesetzmäßigkeiten und weitere neue Folgerungen aus
der Theorie angegeben worden.
Zeitschrift für Physik. Bd. XXXV.
The idea is very deep, and marks the beginning of Quantum Field Theory, in particular Quantum Electrodynamics. The *Umdeutung* procedure of Heisenberg is very general, and provides the correct quantum mechanical formulation valid for a large class of systems. In particular we can consider the classical Maxwell equations, at the beginning in free space, and apply to them the *Umdeutung*. We arrive at the definition of the quantum dynamics of electromagnetic fields. As a matter of facts, the electromagnetic field
in a cavity can be considered as a collection of harmonic oscillators, to be subject to quantization.

It is exactly what it is done in this 3M paper, albeit in a quite synthetic and cryptical way. However, the 3M paper has been recently carefully and deeply scrutinized. All problematic aspects have been clarified, the substantial physical and formal rightness has been recognized.

In the last section of their paper, the Authors study the statistics of these quantum fields
of waves, and find that the distribution in the thermal cavity is in agreement with Planck distribution, moreover the fluctuations are in agreement with the famous Einstein formula.

This reasoning opens the way to a full solution of the dilemma particle-wave for the radiation. The dilemma originates only from the fact that one tries to apply classical concepts to the radiation phenomena. But now it is recognized that the correct description is given by a quantum field theory, which synthetize particle and wave aspects in a single structure.
developments of quantum electrodynamics

After this first strategic conceptual step, the research on quantum electrodynamics continues. We only mention the 1927 contributions by Paul Dirac “The Quantum Theory of the Emission and Absorption of Radiation”, and by Jordan and Wolfgang Pauli “Zur Quantenelektrodynamik ladungsfreier Felder” (On quantum electrodynamics of fields in absence of charges, 1927), where in particular the relativistic commutators of the elec-
Electromagnetic fields in space-time are given for the first time.
Paul Adrien Maurice Dirac
At the 1929 Charkow Conference on Theoretical Physics, organized by D. Ivanenko and V. Fock, Pascual Jordan, a strong promoter of the quantum field theory point of view in physics, gives a review about the status of quantum electrodynamics, then published as “Der gegenwärtige Stand der Quantenelektrodynamik”, on the Physikalische Zeitschrift.

In Rome there is a Great Inquisitor, who is following the scientific mouvement. Let us hear his opinion extracted from a letter to his
friend Giovanni Gentile jr, dated 22-XII-29 - VIII.

Ho letto l’articolo di Gamov che mi hai indicato; mi sembra che dia veramente una buona idea di quelli che sono i primi vagiti della nascente teoria dei nuclei. La quale mi sembra tuttavia che non abbia alcuna probabilità di giungere a maturità se non innestata sul tronco dell’elettrodinamica quantistica, che a sua volta emette ancora i più pietosi vagiti (leggere z.B. un articolo, se non erro di Jordan, sulla “physikalische” del 1° novembre, o giù di lì).
lavori di Fermi, mondi, più necessaria, d'altronde, al disiderio espresso dall'illestr. Kindler, l'unica copy in mio possesso della mia tesi di laurea, benché contenga ben poco di interesa-
tante.

Di tutto l'articolo di Gennaro che mi hai indi- cato, mi sembra che dia veramente una buona idea di quello che vanno i primi sogni dell'e-
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dell'elettrodinamica quantistica, che a sua volta
sente ancora i più pietosi sogni (leggere
26. in contrasto, la vera errore di London, nella
fisica britannica del 10 novembre, o giù di lì)
ma oltre parole il problema dell'aggiungere...
(I read the paper by Gamow that you indicated to me. It seems to me that it truly gives a good idea of what are the first whimpers of the nascent theory of nuclei. However, in my opinion that theory has no probability to reach maturity if it is not grafted on the trunk of quantum electrodynamics, which in turn still emits the most pitiful whimpers (read for example a paper, if I am not mistaken by Jordan, on the “physikalische” of November first, more or less.))
the University booklet of the Great Inquisitor
As a matter of fact there was still a long way ahead, toward the establishment of a complete scheme for quantum electrodynamics. The main problem is with the quantum description of the electrically charged matter interacting with the electromagnetic field. It will be necessary to gain the relativistic formulation of the quantum mechanics of the electron (Dirac 1928), the experimental discovery of the positron (the electron with a negative charge) (Anderson 1932, and Blackett-Occhialini 1933), and then the construction of the so called second quantized
theory of the electron-positrons field, where particles can be created and destroyed. Along this line, there were important contributions by Heisenberg-Pauli 1929-1930, where they for the first time attempt the second quantization of the Dirac field, by Vladimir Alexandrovich Fock 1933, and again by Heisenberg 1934. At the end it is found that quantum electrodynamics can be derived, through an appropriate quantization procedure, from a system of equations containing the coupled Maxwell equations and Dirac relativistic wave equations.
Therefore, starting from 1933-1934 quantum electrodynamics can start its difficult but very successful life.

We would like to end this short review about the developments of quantum electrodynamics with two remarks. The first is about the early involvement of Enrico Fermi in the development of the first stage of quantum electrodynamics (a detailed analysis will appear in a forthcoming paper with Nadia Robotti), which is well synthetized in a well known paper on Reviews of Modern Physics 1932.
QUANTUM THEORY OF RADIATION*

By Enrico Fermi
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INTRODUCTION

Until a few years ago it had been impossible to construct a theory of radiation which could account satisfactorily both for interference phenomena and the phenomena of emission and absorption of light by matter. The first set of phenomena was interpreted by the wave theory, and the second set by the theory of light quanta. It was not until in 1927 that Dirac succeeded in constructing a quantum theory of radiation which could explain in an unified way both types of phenomena. In this article we shall develop the general formulas of Dirac’s theory, and show its applications to several characteristic examples (Part I). In the second part of this work Dirac’s relativistic wave equation of the electron will be discussed in relation to the theory of radiation. The third part will be devoted to the problems of the general quantum electrodynamics, and to the difficulties connected with it.

* Lectures delivered at the Symposium for Theoretical Physics during the Summer Session of 1930 at the University of Michigan.
Fermi methods, as usual, are very simple and direct. A whole generation of physicists learned quantum electrodynamics in the form originally developed by Fermi. Among them Hans Bethe, who visited Rome in 1931. In his own words: “Fermi changed my whole style of doing physics and weaned me from the formal structure of most European universities”.
Enrico Fermi (with bug), Werner Heisenberg, Wolfgang Pauli, at the Como Conference, 1927
Our second remark concerns the Great Inquisitor. After his 1929 sarchastic comments about the “pitiful whimpers” of quantum electrodynamics, may be he became at least partially satisfied in 1933-34 after the weaning produced by the work of Fock and Heisenberg. In fact, he was willing to give his own important contribution through a formulation of quantum electrodynamics completely symmetric between electron and positron. Here is a page of the original incomplete manuscript, kept at the Domus Galileana in Pisa, possibly dated 1934.
Dalla variazione di potenziale elettromagnetico, si deduce allora la seguente espressione per la densità di carica e di corrente:

\[
\begin{align*}
\sigma &= -\varepsilon \left( \nabla \times \mathbf{H} \right) = -\varepsilon \left( \mathbf{E} \times \mathbf{B} \right) \\
\mathbf{J} &= \mu \varepsilon (\mathbf{E} - \varepsilon \mathbf{V} \times \mathbf{B}) = \mu \varepsilon \mathbf{E} - \mu \varepsilon \mathbf{V} \times \mathbf{B}.
\end{align*}
\]

Queste espressioni differenziano le equazioni del moto elettrico, e il campo elettromagnetico si determina all'interno dell'area in cui si applicano. In particolare, in un contesto di trasformazioni e variabilità delle cariche elettriche, l'equazione di Maxwell si rende particolarmente complessa e richiede un approccio dettagliato alla soluzione.
Unfortunately, the manuscript was published only in 1937, under the pressure of a forthcoming professorship competition.
TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE
Nota di Ettore Majorana

Sunto. - Si dimostra la possibilità di pervenire a una piena simmetricizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di Dirac ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di «antiparticelle» correspondenti ai «vuoti» di energia negativa.

L'interpretazione dei cosiddetti «stati di energia negativa» proposta da Dirac (1) conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accordi con il suo contenuto, non sono del tutto soddisfacenti; sia perché si parte sempre da una impostazione asimmetrica, sia perché la simmetricizzazione viene in seguito ottenuta mediante tali procedimenti (come la cancellazione di costanti infinite) che possibilmente dovrebbero evitarsi. Perciò abbiamo tentato una nuova via che conduce più direttamente alla meta.

Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

Its results usually are not recalled as an important contribution to the origins of quantum electrodynamics, as they would be if published in 1934, but only through a by-product of the symmetric formulation contained herein: the possibility to describe neutral particles in a novel way, where particles and anti-particles coincide. As it is very well known, this hypothesis of the Majorana neutrino is still subject to intense research, even at the experimental level.
Let us go back to the situation prevailing in theoretical physics after the “creation” of modern quantum mechanics.
The Schrödinger scheme, the wave function.

Few months after the appearance of the paper by Heisenberg on the Umdeutung, Erwin Schrödinger publishes a series of papers, where he reaches a quantum formulation, along a completely different path.

Schrödinger follows some daring conceptions of Louis De Broglie, according to whom the quantum conditions of Bohr-Sommerfeld must be interpreted as stationarity conditions for
a wave strictly associated to the behavior of the electron. De Broglie discovery points to a wave aspect for matter, together with the well known corpuscular aspect.

By reading the first five papers of Schrödinger, there is a marked sensation of a continuously developing research. At the beginning there is still the dominating idea that the wave function lives in space-time, by providing a kind of “guide” for the behavior of the particle. In particular, the wave function at the beginning is assumed to be real.
In any case, in the frame of this constellation of ideas, Schrödinger reaches the correct formulation of the wave equation.

By exploiting modern notation, the quantum behavior of a particle is described in a complex Hilbert space, the state space, $\mathcal{H}$, made by square integrable configuration functions.

$$\mathcal{H} \ni \psi : R^3 \ni x \rightarrow \psi(x),$$

$$\int dx |\psi|^2(x) < \infty.$$
The quantum evolution is ruled by the Schrödinger equation

\[ i\hbar \partial_t \psi(x, t) = -\frac{\hbar^2}{2m} \Delta \psi + V(x)\psi. \]

In the particular case of stationary states, the Schrödinger equation reduces to the eigenvalue equation

\[ \psi(x, t) = e^{-iEt/\hbar}\psi_0(x), \]

\[ E\psi_0(x) = -\frac{\hbar^2}{2m} \Delta \psi_0 + V(x)\psi_0. \]

The energy levels can be easily calculated,
in full generality, by solving the eigenvalue equation. Some years before, there were similar results by Cornelius Lanczos, recognized only many decades after.

In any case Schrödinger’s result are of absolute historical relevance. The Nobel Prize is awarded in 1933, shared with Paul Dirac, “for the discovery of new productive forms of atomic theory”.

Since Heisenberg wins the Nobel Prize 1932, announced in 1933, while Dirac and Schrödinger
share the 1933 Prize, the three meet together in the same ceremony in Stockholm.
As in any official ceremony, a man MUST be accompanied by a woman. Schrödinger comes with his wife. Heisenberg and Dirac, not married yed, come with their respective mothers. Later Dirac will marry the sister of Eugene Wigner. Heisenberg, strongly attacked in his scientific and political opinions by ultranazist groups (weiss Juden), and ferociously threaten, married in 1937 the beautiful and very blonde Elisabeth Schumacher. Seven blond children were born in few years: Maria, Wolfgang, Barbara, Christine, Jochen, Martin and Verena.
By the work of Dirac, and also Schrödinger, it will be immediately recognized that Heisenberg scheme and Schrödinger scheme are substantially equivalent. In the first, observables evolve in time and the state stays fixed. In the second, the state evolves and observable stay fixed.

However, this is not strictly true, because in Heisenberg theory it is not necessary to introduce the very concept of quantum state, which is not observable in general. Only the operatorial observables are necessary.
The introduction of the notion of quantum state, given by the wave function, is at the origin of all paradoxical features of quantum mechanics.

In a sense, in the Heisenberg theory all states are treated on the same footing. Energy eigenvalues and cross sections are the objects directly connected with the experimental setting.
Epilogue

In recent times, one can notice an increase of interest in the original formulation of quantum mechanics give by Heisenberg.

For example, let us point out the appearance of a very interesting article with a very provocative title “Finally making sense of the double-slit experiment”, appeared in 2017 on the Proceedings of the National Academy of Sciences, with authors: Yakir Aharonov, Eliahu Cohend, Fabrizio Colombo, Tomer
Landsberger, Irene Sabadini, Daniele C. Strupp, e Jeff Tollaksen, and affiliation: Institute for Quantum Studies, Chapman University; Schmid College of Science and Technology, Chapman University; School of Physics and Astronomy, Tel Aviv University; H. H. Wills Physics Laboratory, University of Bristol, e Dipartimento di Matematica, Politecnico di Milano.
In this paper, there is a complete analysis of the quantum double-slit experiment, by relying on the original formulation given by Heisenberg in 1925.

Only the observables position and momentum are involved, in their time evolution.
It is amusing to see that, due to the operator nature, the evolution of the observables is fully sensitive to the case where the two slits are open, with respect to the closure of one slit.

This paper is relevant, because it shows that it is possible to describe in a deep way, in the spirit of Göttingen, quantum phenomena, which show paradoxical features in the wave function picture.
The conclusion of this talk is obvious. The merging of gravitation and quantum mechanics can be achieved through a direct *Umdeutung* of the Einstein-Hilbert equation of motion.

The equations stay the same, but the dynamical variables describing the geometry of space-time acquire an operatorial meaning. There is no wave function of the Universe.

Nontrivial difficulties related to the gauge structure of the geometrodynamical variables
show up, and they must be coped with as in the quantum theory of gauge fields.

A complete account of the emerging theory will be presented in some other occasion.

May be after a necessary visit in Helgoland.