Is there some place for contradictions?  
From paraconsistent logic to Physics

Dominique Lambert  
Université de Namur  
(Département de Philosophie)
A REMARK AND SOME QUESTIONS

We want to speak about the relations between Logic and Physics. But when we are speaking about logic we are speaking of different things:

1) Formal languages and their Models (semantics): Logic is the rules of reasoning
2) Algebraic structures: Boolean rings, Heyting algebras, Brouwer algebras,…: Logic is to operate on algebraic objects
3) Order structures: Boolean lattices, non-commutative lattices, non-distributive lattices, …: Logic is to introduce some order
4) Topological structures: sets, intrinsic logic of some categories (topos,…): Logic is to consider the relative position of objects, or the relative situation of an object with respect to his environment: Logic is to speak about position in a space.

Which facet of logic could be important in Physics?

1) Is Physics forcing us to changing our way of reasoning? « The Max Jammer test! » (The Philosophy of Quantum Mechanics. The interpretations of Quantum Mechanics in Historical Perspective, New York, Wiley, 1974): you can continue to reason in classical logic. PROBLEM: you will speak with a classical language to describe a non-classical ontology
2) 3) 4) If you consider non-classical structures: has this change of logic (of structures) some consequences on the ontology you are using to interpret your results.

If logic is not only a question of reasoning but a question of architecture of structures: what could we said about the emergence of logic? About the link between space, time and logic: what about logic without space and time?
Logical nature of physical theories... not of reasoning!

G. BIRKHOFF, J. von NEUMANN, « Logic of quantum mechanics » 1936
Annals of Mathematics, 37 (4) (1936) 823-843

« The object of the present paper is to discover what logical structure one may hope to find in physical theories which like quantum mechanics, do not conform to classical logic. Our main conclusion based on admittedly heuristic arguments, is that one can reasonably expect to find a calculus of propositions which is formally indistinguishable from the calculus of linear subspaces with respect to set products, linear sums and orthogonal complements – and resembles the usual calculus of propositions with respect to and, or and not »
Logic and Quantum Mechanics


Logic of Quantum mechanics is in fact not at all paraconsistent:
Logic of lattice of closed subspace of a Hilbert space (characterized by projectors):
P is true in a state $\psi$ if $\text{Prob}(p; \psi) = 1 = \langle \psi | P_p | \psi \rangle \Rightarrow P_p | \psi \rangle = | \psi \rangle$,

\[
p \rightarrow \mathcal{H}_p \quad \neg p \rightarrow \mathcal{H}_{\neg p} = (\mathcal{H}_p)_{\perp} \quad p \land q \rightarrow \mathcal{H}_p \cap \mathcal{H}_q \quad p \lor q \rightarrow \text{Closure}(\mathcal{H}_p + \mathcal{H}_q)
\]

The lattice is non-distributive

\[
\mathcal{H}_p \land (q \lor r) \neq (\mathcal{H}_p \cap \mathcal{H}_q) \cup (\mathcal{H}_p \cap \mathcal{H}_r)
\]

(take for example: $\mathcal{H}_q + \mathcal{H}_r = \mathcal{H}$ 2-dim $\mathcal{H}_p \cap \mathcal{H}_q = \mathcal{H}_p \cap \mathcal{H}_r = 0$)

- But: P. Destouches, *La structure des théories physiques*, Paris, PUF, 1951: complementarity (à la Bohr) means certain propositions are incomposables (their "$\land$" does not exist!):
quantum logic has more than two truth values. Furthermore you can get a Paraconsistency Logic of Quantum Superposition! N. C.A. Da Costa, de Ronde, Krause,...
The possibility of using non-standard systems does not necessarily entail that classical logic is wrong, or that (in particular) quantum theory needs at a moment another logic. Physicists probably will continue to use classical (informal logic) in the near future. But we should realize that other forms of logic may help us in better understanding of certain features of the quantum world as well, not easily treated by classical devices, as the concepts of complementarity and of individuality show.

To summarize, we think that there is not just one « true logic », for distinct logical (so as mathematical and perhaps even physical) systems can be useful to approach different aspects of a wide field like quantum theory...

N. C. Da Costa, D. Krause, “Remarks on the applications of paraconsistent logic to physics”,

available on the Web: http://philsci-archive.pitt.edu/1566/1/CosKraPATTY.pdf
Pending
1 Paraconsistent Logic: definition

In Classical Proposition Logic: you have the famous « Ex contradictione sequitur quodlibet » (ECQ)

\[ A \land \neg A \Rightarrow B \quad (B \text{ is an arbitrary proposition}) \]

In Classical Logic: contradictions are « explosive » they are destroying all the deductive system, implying all what you want!

=> There is no (classical) model for deductive system denying ECQ

You can imagine a Logic that tolerates some contradictions (but not all!)

=> Could we propose a model of such situation?

To what situations corresponds the abandon of ECQ?
2 A topological classical example
Let us remember the link between classical logic and topology
Classical Logic $\Rightarrow$ Sets $\Rightarrow$ Category of sets

If you want to change your logic you have to change your category…. Let us choose a category which is not the category of sets: a general topos for example.

THEN YOU CHANGE YOUR NOTION OF SEPARABILITY...OR THE LINK BETWEEN SYSTEMS AND ENVIRONMENT
Intuitionistic Logic $\Rightarrow$ Open sets $\Rightarrow$ Topos (category of sheaves: generalized notion of space)

Paraconsistent Logic $\Rightarrow$ Closed sets $\Rightarrow$ ????

When changing logic, you are changing the border.... The relation between objects and environment; the notion of space? SPACE AS SHEAF; SHEAF AND LOGIC?
**CLASSICAL LOGIC**

\[ p \models \Box A_p \quad A_p \Box P(E) \]

\[ p \models \Box q \quad \Box (A_p) \cdot A_q \]

\[ p(q) \models \Box A_p^* \cdot A_q \]

\[ \neg p \models \Box C E( A_p) \]

\[ p \models \neg p \models \Box C E( A_p) \Rightarrow = E \]

\[ p \wedge \neg p \models \Box \cdot C E( A_p) \Rightarrow = \emptyset \]

---

**INTUITIONISTIC LOGIC**

\[ p \models \Box U_p \quad U_p \Box O(E) \]

\[ p \models \Box q \quad \Box U_p \cdot U_q \]

\[ p(q) \models \Box U_p^* \cdot U_q \]

\[ p(\neg p) \models \Box U_p^* \cdot \Box C E(U_p) \neq E \]

\[ p \models \neg p \models \Box \cdot \Box C E(U_p) \Rightarrow = \emptyset \]

---

**PARACONSISTENT LOGIC**

\[ p \models \Box W_p \Box F(E) \]

\[ p \models \Box q \quad \Box W_p \cdot W_q \]

\[ p(q) \models \Box W_p^* \cdot W_q \]

\[ p(\neg p) \models \Box W_p^* \cdot \Box C E(W_p) \Rightarrow = E \]

\[ p \models \neg p \models \Box U_p \cap \Box C E(U_p) \neq \emptyset \]

---

**BOOLEAN ALGEBRA**

**CATEGORY OF SETS**

**HEYTING ALGEBRA**

\[ (p \wedge q) \leq r \iff p \leq (q \Rightarrow r) \]

**TOPOS (category of sheaves)**

**COMPLEMENT TOPOS**

**CO-HEYTING ALGEBRA**

\[ (p-q) \leq r \iff p \leq (q \lor r) \]

---

**CO-HEYTING ALGEBRA**

\[ (p-q) \leq r \iff p \leq (q \lor r) \]

\[ p \models \Box W_p \Rightarrow F(E) \]

\[ p \models \Box q \quad \Box W_p \cdot W_q \]

\[ p(q) \models \Box W_p^* \cdot W_q \]

\[ p(\neg p) \models \Box W_p^* \cdot \Box C E(W_p) \Rightarrow = E \]

\[ p \models \neg p \models \Box U_p \cap \Box C E(U_p) \neq \emptyset \]
LOGIC IS ALSO ABOUT PROCESSES
CATEGORY DESCRIBES PROCESSES: « objects and arrows philosophy »: link Category-Physics

For example: see the work about CATEGORICAL QUANTUM MECHANICS


THERE ARE VARIOUS LINKS BETWEEN CATEGORY AND LOGIC

- A logic can be implemented in some particular category (cartesian closed, monoidal, symmetric, braided,...)

- A logic can be intrinsic to a category (classical logic and category of sets, intuitionnistic logic and elementary topos,...)

- Category can be considered as a category of model of some formal languages (see classifying topos: Caramello, Lafforgue)

WHAT COULD BE THE LINKS BETWEEN PARACONSISTENT LOGIC AND CATEGORIES?

AND PHYSICS? ARE THEY FRUITFUL?
PHYSICS => PROCESSES => MONOIDAL CATEGORY => LOGIC

<table>
<thead>
<tr>
<th>Category Theory</th>
<th>Physics</th>
<th>Topology</th>
<th>Logic</th>
<th>Computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>object</td>
<td>system</td>
<td>manifold</td>
<td>proposition</td>
<td>data type</td>
</tr>
<tr>
<td>morphism</td>
<td>process</td>
<td>cobordism</td>
<td>proof</td>
<td>program</td>
</tr>
</tbody>
</table>

```
A
\downarrow
B
```

```
A \downarrow \quad A
```

```
\quad C \otimes D \downarrow \quad A
```

```
\quad \cap \downarrow \quad \cup
```

```
\quad A
```

« A TOPOS FOUNDATION FOR THEORIES OF PHYSICS: I. FORMAL LANGUAGES FOR PHYSICS »
A. Doering, C.J. Isham

« Physics, Topology, Logic and Computation: A Rosetta Stone »
John C. Baez, Mike Stay,
- Cartesian Closed Categories (Lambeck) - Classical Logic (Boolean algebras)
- Monoidal Categories (category of representation) - Classical Logic (Boolean algebras)
- Closed monoidal symmetric categories (Girard, Ambler) - (multiplicative intuitionistic) Linear Logic
- Topos (category of sheaves on M) (Suszko, Bell,...) - Heyting Algebras

*Paraconsistency is introduced here*

- **Complement Topos** (Mortensen, Lavers, James) - Brouwer (co-Heyting algebras)


PHYSICS: possible sources of paraconsistent concepts

Continuum Physics
- F.W. Lawere, S.H. Shanuel
- S. Majid, J. Petitot

Superposition in Quantum Mechanics
- Da Costa, de Ronde, Krause,... Paraconsistency Logic of Quantum Superposition!

Paraconsistent probability theory

Paraconsistent Quantum Logic
- Effects Algebras (generalization of projectors), Brouwer-Zadeh Posets,...

Indefinite metric space (Krein Spaces,...) ?

Weak measurements ?
Closed sets topology and paraconsistent logic: the algebraic way


PARACONSISTENT LOGIC, CHANGE AND BORDER
**Continuum Physics and the birth of geometry?**


The border! Contradictions are restricted to the border and do not invade all the system

Contradiction and the Leibniz rule
The origin of derivation operator (geometry?)

\[
(p \land \neg p) \iff \partial p
\]

\[
\partial (p \land q) \iff (\partial p \land q) \lor (p \land \partial q)
\]

Logic and the birth of geometry?
Let us use the CLOSED sets (Like Leray in his first sheaf theory)

\[ \partial p = p \land \neg p \]  

this is the border!!!  

\[ \partial W_p := W_p \cap \sim W_p \]

\[ \neg p \mid \square \text{ClosCE} (W_p):= \sim W_p \]

\[ W_p = \sim \sim W_p \cup \partial W_p \]

The study of the border of the closed sets began

As Lawere noted in 1927 with the works of ZARICKI and afterwards in the famous book of Topology of Kuratowski

\[ \partial ( W_p \cap W_q ) = (\partial W_p \cap W_q ) \cup (W_p \cap \partial W_q ) : \text{the Leibniz rule:} \]

\[ \partial (f.g) = \partial f \cdot g + f \cdot \partial g \]

\[ \partial ( W_p \cup W_q ) \cup \partial(W_p \cap W_q ) = \partial W_p \cup \partial W_q \]

Continuum physics, thermodynamics, etc.
How to conceive and to describe change by contradiction? It means the transition from a state $p$ to a stat NOT-$p$?

This is a classical philosophical problem:

cfr Aristotle, Physics, VI, 5, 235b.:

« Since everything that changes changes from something to something, that which has changed must at the moment when it has first changed be in that to which it has changed.

For that which changes retires from or leaves that from which it changes: and leaving, if not identical with changing, is at any rate a consequence of it. And if leaving is a consequence of changing, having left is a consequence of having changed: for there is a like relation between the two in each case.

One kind of change, then, being change in a relation of contradiction, where a thing has changed from not-being to being it has left not-being. Therefore it will be in being: for everything must either be or not be. It is evident, then, that in contradictory change that which has changed must be in that to which it has changed. »
See the famous comments of St Thomas Aquinas of the Aristotle’s book of Physics

[72193] In Physic., lib. 5 l. 3 n. 3 Deinde cum dicit: secundum substantiam autem etc., manifestat conditionalem praemissam. Et primo ostendit quod in aliis generibus a tribus praedictis, non potest esse motus; secundo ostendit quomodo in istis tribus generibus motus sit, ibi: quoniam autem neque substantiae et cetera. Circa primum tria facit: primo ostendit quod in genere substantiae non est motus; secundo quod nec in genere ad aliquid, ibi: neque est in ad aliquid etc.; tertio quod nec in genere actionis et passionis, ibi: neque agentis neque patientis et cetera. Praetermittit autem tria praedicamenta, scilicet quando et situm et habere. Quando enim significat in tempore esse; tempus autem mensura motus est: unde per quam rationem non est motus in actione et passione, quae pertinent ad motum, eadem ratione nec in quando. Situs autem ordinem quendam partium demonstrat; ordo vero relatio est: et similiter habere dicitur secundum quandam habitudinem corporis ad id quod ei adiacet: unde in his non potest esse motus, sicut nec in relatione. Quod ergo motus non sit in genere substantiae, sic probat. Omnis motus est inter contraria, sicut supra dictum est: sed substantiae nihil est contrarium: ergo secundum substantiam non est motus.

Let have a look also on:

St Thomas Aquinas: Comm. In IV Sent. D.11, q.1, a.3, qc 2, ad 2

What Lawvere and Paraconsistent Logic shows is another way to conceive « change by contradiction »

**Classical one:**

You have no last time in which X is not-p but you have a first instant where X is p

**Paraconsistent way of thinking**

δ p = p ∧ ¬ p

δ p is a « derivative »: change between p and ¬ p

Paraconsistency offers a way to think about a mediation between p and ¬ p

CONTRADICTION IS LOCALIZED (non sequitur quodlibet!)
COBORDISM:
\[ \partial p = p \land \neg p \]

CLASSICAL LOGIC
Change by contradiction is a discontinuous transition

\[ \neg p \]

\[ p \]

PRACONSISTENT LOGIC
Change by contradiction reflects singular or unstable state

\[ \neg p \]

\[ \partial p = p \land \neg p \]

COBORDISM: \[ \partial p = p \land \neg p \]
A unified pre-conciliation $G$ in $X$ is a **conciliation** in $X$ iff it satisfies the following property:

for every $W \in \text{Fer}(X)$, $W \neq \emptyset$,
every finite closed co-covering $(W_i)_{i \in I}$ of $W$,
and every $(t_i)_{i \in I}$ such that $\forall i \in I : t_i \in G(W_i)$,
if $\forall i, j \in I : \delta^W_{W_i \cup W_j}(t_i) = \delta^W_{W_j}(t_j)$
then $\exists! t \in G(W) \forall i \in I : \delta^W_{W_i}(t) = t_i$. (4)

\[ \text{Lim } G(W_k) = \text{F}(\cap W_k) \]

\[ \text{Lim } G(W_k) = \text{F}(\cup W_k) \]

A dualization of Sheaves:

Category of conciliation

$\forall W \in \text{Fer}(X)$, $W \neq \emptyset$,
\[ \delta^W_{W_i \cup W_j}(t_i) = \delta^W_{W_j}(t_j) \]
\[ \delta^W_{W_i \cap W_j}(t) = t_i. \]

\[ \text{Lim } G(W_k) = \text{F}(\cap W_k) \]

\[ \text{Lim } G(W_k) = \text{F}(\cup W_k) \]

D. Lambert, B. Hespel, “From contradiction to conciliation: a way to "dualize" sheaves”, arXiv:1106.6194v2


Are there some other places in physics where you could meet paraconsistent objects?
What could you say if

$$\mathcal{G}_{\neg p} = (\mathcal{G}_p)\perp \text{ or } \supseteq \mathcal{G}_p$$

Neutral subspaces.

They do not exist with Hilbert spaces... OK but they exist with Krein Spaces (indefinite metric spaces)


$$\mathcal{G}$$ is a neutral subspace of a Krein space $$\mathcal{H} = \mathcal{H}^+ \bigoplus \mathcal{H}^-$$

$$\mathcal{G}_p\perp = \mathcal{G}_p \bigoplus \mathcal{G}^+ \mathcal{G} + \mathcal{G}^- \mathcal{G}$$ ★ you get paraconsistency!
Indefinite metric space are a little bit strange but well-known in the mathematical literature.

They are related to signed measure theory: that can defined negative probabilities!

Negative probabilities arise in Physics:

1) in Weak measurement theory (Wigner transform is a quasi-probability distribution taking negative values!

2) In quantum gravity: an old example given by Vilenkin

\[ ds^2 = -N^2 dt^2 + a^2(t) (dx^2 + \sin^2 \theta \, d\theta^2 + \sin^2 \phi \, d\phi^2) \]

\[ \rho_\phi (a) = \frac{1}{2} \, i \alpha^p (\psi^* \partial_a \psi - \psi \partial_a \psi^*) \]

\[ \psi = \exp(i\pi 4) (a^2 V - 1)^{-1/4} a^{-p+1/2} \exp (- (1+i (a^2 V - 1)^{-3/2} /3V) \) solution of the Wheeler-de Witt equation \( H \psi(a, \Phi) = 0 \)

The wave function corresponding to an expanding universe corresponds to positive probabilities.
The wave function corresponding to an contracting universe corresponds to negative probabilities.

Question: is there some link between Paraconsistent logic and Negative probabilities?
What could its physical meaning? Paraconsistencies and singular, discontinuous, unstable situations?
Paraconsistent probability theory


One allows contradiction to get non-zero probability: you

\[ P(b) = P(b \land a) + P(b \land \neg a) - P(b \land a \land \neg a) \]

\[ P(a \mid b) = \frac{P(b \mid a) \cdot P(a)}{P(b \mid a) \cdot P(a) + P(b \mid \neg a) \cdot P(\neg a) - P(b \mid a \land \neg a) \cdot P(a \land \neg a)} \]

You can also define a paraconsistency probability space
Paraconsistent Quantum Logic and their algebras (effects algebra)


Sharp approach: fuzzyness related to the state
Events: \( P^+ = P \quad P^2 = P \) projector
State : density matrix \( \rho \) => Gleason Theorem: Probability measure \( \omega_\rho(P) = \text{Tr} (\rho \ P) \in \mathbb{C} \) for any density matrix \( \rho \)

Unsharp approach: you considered fuzziness related to the accuracy of the measurement
Projectors \( \rightarrow \) Effects \( E \) : bounded linear operator \( \text{Tr} (\rho \ E) \in \mathbb{C} \) for any density matrix \( \rho \)

intuition: « sharp » : ambiguous state, no ambiguity on the measured property
« unsharp »: ambiguous state and ambiguous measured property
The set of all effects can be endowed with a structure of Brouwer-Zadeh Poset

$$(\mathcal{E}(\mathcal{H}), \leq, ', ~, \emptyset, 1)$$ is a Brouwer-Zadeh Poset on a Hilbert space $\mathcal{H}$

$E \leq F$ iff $\text{Tr} (\rho^{} E) \leq \text{Tr} (\rho^{} F)$ for all density matrix $\rho$

First negation: $E' = 1 - E$

Second negation: $E \sim = P_{\ker(E)}$ the projector on the kernel of $E$

$\emptyset$: is the null projection ; $1$: is the identity projection

In this structure, the noncontradiction principle is violated: $E \land E' \neq \emptyset$
Paraconsistent logic could be not a new logic of reasoning

but

+ a way of characterizing the operator algebras used in quantum mechanics

+ a way of characterizing quantum objects (a logic for a quantum ontology?)
LES PROBLÈMES DE LA PHILOSOPHIE DES SCIENCES

L'HYPOTHÈSE
DE L'ATOME PRIMITIF

ESSAI DE COSMOGONIE

PAR
GEORGES LEMAITRE
PROFESSEUR À L'UNIVERSITÉ DE LOUVAIN

PRÉFACE DE FERDINAND CONSEIL
PROFESSEUR À L'ÉCOLE POLYTECHNIQUE FÉDÉRALE

NEUCHATEL
ÉDITIONS DU GRIFFON

BRUXELLES
ÉDITIONS HERMES

Société Helvétique des Sciences naturelles
Fribourg, 1er-3 septembre 1945
“Le commencement (de l’univers) se situe juste avant le commencement de l’espace et du temps (...) Il se situe juste avant la Physique. C’est le fondement inaccessible de l’espace-temps. Une telle image trouve un support géométrique naturel dans la singularité ponctuelle qui se présente dans la théorie de Friedmann. Le rayon de l’espace peut partir de zéro. Un événement aussi singulier se présente quand l’espace a un volume nul. C’est le fond de l’espace-temps. Je ne prétends pas qu’une telle singularité est inéductable dans la théorie de Friedmann, mais je montre simplement comment elle s’adapte au point de vue quantique comme un commencement naturel de la multiplicité et de l’espace-temps”.

(Congrès Solvay 1958, La structure et l’évolution de l’univers, Bruxelles, Stoops, 1958, p.7)

The beginning of the universe is located just before the beginning of space-time!

You can represent the beginning of space-time from a state that is logically anterior (algebra): algebra can produce time evolution....

Logic as properties of structures could pre-exists to space-time notions (this could be the logic of primeval atom!)
QUATERNIONS ET ESPACE ELLIPTIQUE

GEORGES LEMAÎTRE

SYNOPSIS. – Au lieu que le quatrième du nombre quaternion soit absorbé dans le nombre unité, nous l’avons maintenu dans un espace bi-tridimensionnel, pourvu que toute espace gazeux soit transformé en espace euclidien

1. – INTRODUCTION

Les quaternions ont été inventés en 1843 par Sir William Rowan Hamilton. Il est difficile d’imaginer avec quel enthousiasme, mais aussi avec quelle confusion, cette idée géniale a été développée par son auteur.

Dans une « Introduction to quaternions » publiée à Londres (Mac Millan 1873) par P. Kelland et P. G. Tait, le premier des auteurs déclare : « The first work of Sir W.R. Hamilton Lectures on Quaternions (1852), was very thinly and imperfectly understood by me and I dare say by others ». Il ajoute que les Elements of Quaternions (1865) et même l’exposé plus clair de son co-auteur P. G. Tait, Un Elementary Treatise on Quaternions ne peuvent être considérés comme élémentaires.

Le livre lui-même dont ces remarques sont tirées a certainement un caractère élémentaire, il est vrai même dans ce sens, en présentant des démonstrations de théorèmes trop connus pour lesquels l’emploi d’un nouveau type de calcul ne semble pas se justifier.

Q = q^0 + q^1 i + q^2 j + q^3 k  \quad i.j = - j.i = k

QQ^* = 1 = (q^0)^2 + (q^1)^2 + (q^2)^2 + (q^3)^2

Stereographic projections

Usual Sphere: S^2  Spherical Universe : S^3
Spinors and Quantum Physics
January 1956

G. Lemaître and the Spinor theory

Elie Cartan
Jacques Tits

G. Lemaître: SO(3,3) = SL(4,R)
Dirac-Eddington like equation
1 Logic has a fundamental facet (logic of classical reasoning) but

2 Logic has also an algebraic and topological facet: logic of « structures »

3 Logic has also an empirical facet (As Ferdinand Gonseth said):

Paraconsistent logic as well as fuzzy logic could be considered not as fundamental structure of reasoning in physics (in this case we would have to change classical logic into another!) but as a way to characterize some particular (operatorial) structures corresponding to particular empirical fields. This situation is already present in applied sciences (in robotics for example): where we are using fuzzy logic and paraconsistent logic in the case where informations are fuzzy and inconsistent.

You don’t have to change the logic at the level of your reasoning, but you have to introduce it as an empirical necessity!
A Paraconsistent Logic Program Based Control for a Discrete Event Cat and Mouse

Kazumi Nakama3, Rynji Ishikawa, and Atsuyuki Suzuki2

1School of H.S.E., University of Hyogo, HIMEJI 670-0092, Japan
nakamatsu@hse.w-hyogo.ac.jp
2Dept. Information, Shizuoka University, HAMAMATSU 432-8011, Japan
{ce0005,suzuki}@cs.inf.shizuoka.ac.jp

Abstract. We have developed a paraconsistent logic program called an Extended Vector Annotated Logic Program with Strong Negation (abbr. EVALPSN), which can handle defeasible deontic reasoning and contradiction, and applied it to safety verification and control such as railway interlocking safety verification, traffic signal control etc. In this paper, we introduce how to apply EVALPSN to discrete event control with taking an example called Cat and Mouse. Generally, event control can be represented as deontic rules such as it is forbidden for both the cat and the mouse to occupy the same room simultaneously, and the control must deal with contradiction to avoid unexpected system states. We show that such a discrete event control can be easily formalized in EVALPSN and implemented.

Keywords: paraconsistent logic program, discrete event control, defeasible deontic reasoning, EVALPSN.

References

What is the status of Paraconsistent Logic in Physics?

1 Maybe it has not a fundamental one (classical logic always plays a fundamental role)

2 But it could plays a very important role: this logic expresses properties of structures

   + of some operator algebras, lattices, etc. used in particular fields of physics. It is not a
     property of reasoning! Cfr Dalla Chiara, Giuntini...

   + of some topological situations: where you have some border, some singularity, discontinuity
     surfaces, instabilities? Cfr for example: Lawere, Jean Petitot,...

     Beginning in cosmology, measurement in quantum mechanics,...

If one wants to understand the transition between the one and the many, between potentialities and actualites, one has to face some « change by contradictions »:

the formalism has to express that and maybe paraconsistent logic is a way to track in the algebraix ans topological formalism the trace (vestigium) of such transition?
Some references:

-D. LAMBERT, B. HESPEL, “From contradiction to conciliation: a way to "dualize" sheaves”, arXiv:1106.6194v2

